**Unit 3: An Analytic Approach**

Goals/Rationale

In this unit students learn the analytic technique of separation of variables and explore analytic and graphical connections. The unit begins by contrasting two approaches to sketching solutions on a slope field in order to maintain strong connections between graphical and analytical techniques. Several new terms are introduced in the unit (exact solution, general solution, particular solution, and initial value problem). Students will need a transparency for problems 7 and 8.

**Page 3.1 – Comparing Predictions**

Implementation Notes

*Problem 1* – This problem asks students to compare two different ways of using a slope field to obtain graphs of solution: the tip to tail method and what is referred to as the “continuously changing rate of change” method. The latter of which is associated with an exact solution. Some suggested discussion questions:

* How do the slopes of the two graphs compare at t = 0? Do they share the same slope? Does whether or not they share the same slope depend on the increment size of the tip to tail method?
* Under what circumstances will there be no difference between Tom and Jerry’s graphs?
* Does the tip to tail graph lie above or below the other graph? Why?

**Pages 3.2 – 3.5 – Separation of Variables**

*Problems 2-5* – Students are guided through the technique of separation of variables, first by using the chain rule and then by using what is referred to as a “short cut” where dy and dt are separated. Many texts only present the short cut, but doing so goes against the spirit of the materials because it hides underlying connections and ideas. Both techniques are presented in a two-column format where the left side states what is to be done and the right side is left blank for students to fill in. This two column format is used to teach other techniques that are either not viable or would take too much time for students to reinvent. These tables can be completed individually, in small groups, or as a whole class discussion. In problem 2 (c) students are told to carry out the steps assuming P > 0. This is done to avoid getting bogged down with ln(|P|). We leave it to the discretion of the instructor to go through the details of this if desired.

*Problem 6* – Students are presented a problem that requires them to pay close attention to issues of domain and to make connections between earlier problems that required them to symbolically check if a function is a solution to a differential equation (does it “fit” or satisfy the differential equation) and to the graphical analog of a function “fitting” the slope field.

**Pages 3.6-3.7 – Making Connections**

*Problems 7-9* – **Transparencies are needed for every student**. These problems are designed to help students make connections between the general solution and solutions as sketched on a slope field. One connection is that both show that there are an infinite number of functions that solve a differential equation, which is not always obvious for students. Another, more subtle connection is how the “+ c” in the general solutions corresponds to the fact that solution graphs that are horizontal shifts of each other along the t-axis for autonomous differential equations and to the fact that graphs of solutions are vertical shifts of each other for differential equations that depend explicitly only on time (e.g., dh/dt = -t +1). Finally, this problem is intended to help students think about the space of solution functions as a “chunk.” For example, for an autonomous DE we want students to be able to think about the entire collection of solution functions as being represented by one or more solution graphs (where all other solutions graphs are simply a horizontal shift). Such thinking will be important for subsequent work with phase lines in Units 6 and 8.

A good way to get started with this problem is to model for students (use transparency on a doc cam) what they are supposed to be thinking about for part (c) – For example, “How can I move the graph of this solution so that it is the graph of the solution with this other initial condition? Should I take this curve and move it up and to the left? Maybe shift it upward?

Before discussing the answer, it is important that each student get a chance to physically move the curve on the transparency because physically moving the graph, having that bodily connection with the space of solutions, is helpful for mentally constructing the space of solutions as an entire chunk. In the past students have told us that they “knew” the graphs should be shifts along the t-axis but that it wasn’t until after they physically did the shift did they really understand why it is a shift.

**Notes for Personal Reflections on Unit 3**